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THE FORCE AND MOMENT ON A SLENDER BODY OF
REVOLUTION MOVING NEAR A WALL   C.—C. 1

by

J. Nicholas Newman

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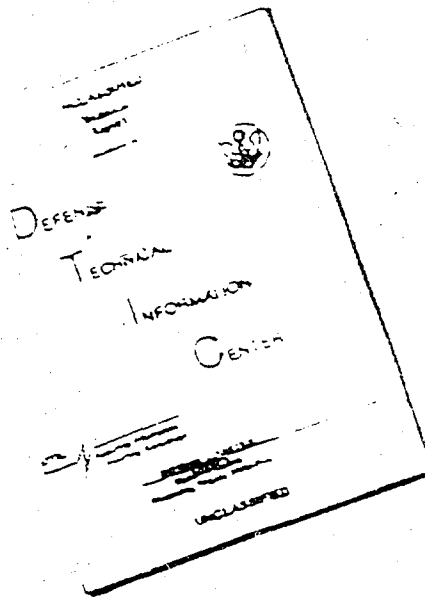
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December 1965   Report 2127

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THE FORCE AND MOMENT ON A SLENDER BODY OF

REVOLUTION MOVING NEAR A WALL

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NOTATION

a	$a = [z_o^2 - r_0^2(x)]^{1/2}$
F_z	Vertical force acting on body
$f(x)$	Arbitrary function
i	Unit vector in x-direction
L	Body length
M_y	Pitch moment acting on body
n	Unit normal vector, positive into the fluid
r	Polar radius coordinate
$r_o(x)$	Body radius as function of x
r_{om}	Maximum body radius
$S(x)$	Sectional area, $S(x) = \pi [r_o(x)]^2$
U	Forward velocity of the body
V	Velocity vector of the fluid
(x,y,z)	Cartesian coordinates (cf. fig. 1)
z_o	Height of body axis above the wall
ζ	$\zeta = z_o / r_{om}$
ξ	Dummy variable corresponding to the coordinate x
θ	Polar angular coordinate
ρ	Fluid density
ϕ	Velocity potential
ϕ_{2D}	Two-dimensional velocity potential

ABSTRACT

Analytical results are derived for the vertical force and pitch moment acting on a body of revolution which is moving with constant forward velocity parallel to an infinite horizontal wall. It is assumed that the fluid is ideal and incompressible and that the body is slender and situated close to the wall. The force and moment are given in terms of simple integrals involving the body shape and separation distance from the wall. Calculations are made for a spheroid, and it is shown that the vertical force rises rapidly as the body is brought progressively closer to the wall.

INTRODUCTION

It is well known in naval architecture that when a body moves through a fluid parallel to a wall, an interaction force and moment will act on the body because of the presence of the wall. Normally the force will be positive in the direction toward the wall, due to the venturi effect of the accelerated flow between the body and the wall, and this force will become more pronounced as the body is moved closer to the wall. The moment will generally increase also, under these circumstances, but its sign is not definite and will depend on the detailed shape of the body; for a body with fore-and-aft symmetry the moment will vanish, under the assumptions of ideal flow, because of the corresponding symmetry of the resulting flow.

There are several instances in which this problem may be of practical interest, particularly for a ship moving in restricted water near a bank or wall of a canal (and where at low speeds the effects of the free surface are not important). Recent interest in oceanographic submarines has posed a similar problem for a vehicle moving in close proximity to the bottom.

To obtain a complete analytical prediction for the force and moment acting on a body near a wall, it is necessary to idealize the problem. With this goal in mind we shall assume not only that the fluid is ideal, incompressible, and unbounded except by the wall and the body, but also that the body is axisymmetric, slender, and in close proximity to the wall. The

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last assumptions permit us to use the slender-body approach of aero-dynamics[1] to obtain an explicit solution of the boundary-value problem for the velocity field, in terms of a source distribution inside the body plus a corresponding image source system below the wall. The strength of this source distribution is unchanged by the presence of the wall, but the effect of the wall is not only to introduce the image system, but also to require that the original source distribution be offset from the body axis toward the wall; this can be inferred from the corresponding two-dimensional problem in bipolar coordinates. If the body were not axisymmetric, a more complicated singularity distribution would be required, and this could only be obtained from a numerical scheme. Nevertheless, it is felt that the present results should be valid for bodies which do not depart too much from axial symmetry, as in the case of a submarine.

Having found the velocity potential, in terms of a three-dimensional source distribution within the body and its image, we can easily find the force and moment acting on the body by the use of Lagally's theorem,[2] and the final results take the form of integrals over the body length involving the cross-sectional area of the body and its first derivative, and the distance of the body from the wall. These results are illustrated for a slender spheroid.

Several years ago Eisenberg[3] considered the same problem for a spheroid, and although his results were limited to finding the pressure distribution on the body. the force and moment could be found therefrom by integration. However, there is a significantly different assumption in Eisenberg's analysis, from those made here, in that the flow is approximated by that due to a single spheroid in an infinite fluid plus the corresponding disturbance due to an image spheroid, without regard for the fact that there will be an induced velocity on each spheroid due to its image which must be accounted for. In fact Eisenberg's approximation will be valid provided the two spheroids are far apart, whereas ours, based upon the

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[1] References are listed on page 10.

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lender-body theory, is valid provided the two bodies are close together.
: is felt that the latter is more appropriate in the present context,
.nce the force and moment under consideration increase rapidly as the
)dy is brought closer to the wall and are comparatively negligible if
ιe body axis is greater than one or two diameters from the wall.

THE VELOCITY POTENTIAL

We shall employ Cartesian coordinates (x,y,z), with $z = 0$ the plane
the wall and with the body axis on the line $y = 0$, $z = z_o$. Thus the
dy axis is parallel to the x-axis and is a distance z_o from the wall
igure 1). The body is assumed to be defined by a smooth continuous
dius function $r_o(x)$, according to the equation

$$r_o(x) = [y^2 + (z - z_o)^2]^{1/2}, \text{ for } -\frac{L}{2} < x < \frac{L}{2}, \qquad [1]$$

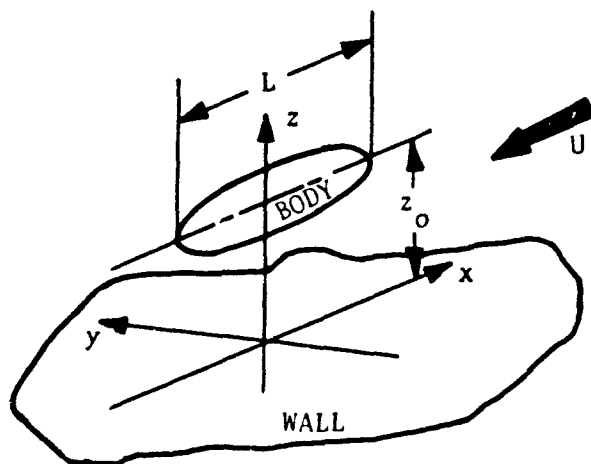

Figure 1 - The Coordinate System and Flow Geometry

re L is the body length and by assumption $r_o(\pm \frac{L}{2}) = 0$. Because of the
nder body assumption and because the wall is in the near field,
L << 1 and z_o/L << 1.

The body is considered to be fixed in space, with a free stream of velocity U flowing past it in the -x direction. The velocity field is defined by the vector

$$\mathbf{V} = \nabla \phi(x,y,z) - U\mathbf{i} \qquad\qquad [2]$$

where ϕ is the velocity potential of the perturbation flow. This potential must satisfy Laplace's equation, the boundary condition

$$\frac{\partial \phi}{\partial z} = 0 \quad \text{on the wall } z = 0,$$

and the boundary condition

$$\frac{\partial \phi}{\partial \mathbf{n}} = U \cos(\mathbf{n},x) = - U\, r_0{}'\,(x)$$

on the body. Here $\mathbf{n}$ is the unit normal vector out of the body and $r_0{}'(x) = dr_0/dx$. In addition, $\nabla \phi$ must vanish at infinity. In the near field, close to the body, it follows from slender body theory[1] that

$$\phi(x,y,z) = \phi_{2D}(y,z;x) + f(x), \qquad\qquad [3]$$

where ϕ_{2D} is the potential of a two-dimensional flow in the "cross-flow" plane. The function $f(x)$ can be found, but since it depends only on x it does not contribute to the vertical force or to the pitch moment.

The potential ϕ_{2D} must satisfy the body boundary condition

$$\frac{\partial \phi_{2D}}{\partial \mathbf{n}} = - U\, r_0{}'(x). \qquad\qquad [4]$$

Physically this is the velocity potential for the flow of fluid out (or in) through the periphery of the circle $y^2 + (z - z_0)^2 = r_0{}^2$, in the presence of a wall at $z = 0$. By reflection this is equivalent to the flow out of (or in through) two circles with centers at $z = \pm z_0$. To solve this two-dimensional problem, we look for a solution of the form*

_____

Some motivation for attempting to find a solution of this form can be found from the properties of bipolar coordinate systems.

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$$\phi_{2D} (y,z;x) = C \log \left\{ [y^2 + (z-a)^2] \ [y^2 + (z + a)^2] \right\}, \qquad [5]$$

ıere C and a are constants (which may depend on x). Physically this ɔtential represents the flow due to a source at $y = 0$, $z = a$, plus an nage source at $y = 0$, $z = -a$. Changing to polar coordinates (r, θ), with ıe origin at the center of the original circle, gives

$$y = r \sin \theta$$

$$z = z_o + r \cos \theta,$$

1 it follows that

$$\phi_{2D} (r,\theta;x) = C \log \left\{ (r^2 + 2 z_o r \cos \theta + z_o^2 + a^2)^2 \right.$$
$$\left. - 4 a^2 (z_o + r \cos \theta)^2 \right\}.$$

can be easily verified that, on the circle $r = r_o(x)$,

$$\frac{\partial}{\partial r} \phi_{2D} = 2 C/r_o, \qquad [6]$$

ovided the parameter a is chosen to satisfy the relation

$$a^2 = z_o^2 - r_o^2.$$

om the boundary condition [4] it follows that

$$C = - \frac{1}{2} U r_o(x) r_o'(x) = - \frac{1}{4\pi} U S'(x),$$

ere $S(x) = \pi r_o^2(x)$ is the cross-sectional area and $S'(x) = d S(x)/dx$. us the desired two-dimensional potential is

$$\phi_{2D} (y,z;x) = - \frac{1}{4\pi} U S'(x) \log \left\{ [y^2 + (z-a)^2] \ [y^2 + (z + a)^2] \right\} \qquad [7]$$

th

$$a(x) = \left\{ z_o^2 - [r_o(x)]^2 \right\}^{1/2}$$

The three-dimensional source distribution which is associated with ϕ_{2D} is

$$\phi(x,y,z) = \frac{1}{4\pi} U \int_{-\frac{L}{2}}^{\frac{L}{2}} S'(\xi) \left\{ [(x-\xi)^2 + y^2 + (z-a)^2]^{-1/2} \right.$$

[8]

$$\left. + [(x-\xi)^2 + y^2 + (z+a)^2]^{-1/2} \right\} d\xi,$$

where in the integral the parameter a is a function of ξ. It can be shown[1] that this three-dimensional potential reduces to ϕ_{2D}, as given by Equation [7], plus a function only of x, for points on or near the body surface.

THE VERTICAL FORCE AND PITCH MOMENT

It is now a straightforward matter to find the vertical force F_z and pitch moment M_y from Lagally's theorem.[2] For this purpose we note from [8] that the body is generated by a distribution of three-dimensional sources, of strength

$$\frac{1}{4\pi} U S'(x),$$

in the presence of an "external flow" due to the free stream of velocity - Ui, where i is a unit vector in the x-direction, and an additional external flow due to the image potential

$$\frac{1}{4\pi} U \int_{-\frac{L}{2}}^{\frac{L}{2}} S'(\xi) [(x-\xi)^2 + y^2 + (z + a)^2]^{-1/2} d\xi.$$

From Lagally's theorem the vertical force acting upon the body is the integral of the differential force on each source element, which in turn is equal to $4\pi\rho$ times the product of the source strength and the vertical component of the external flow. Thus

$$F_z = \frac{1}{4\pi} \rho U^2 \int_{-\frac{L}{2}}^{\frac{L}{2}} S'(x) \left\{ \frac{\partial}{\partial z} \int_{-\frac{L}{2}}^{\frac{L}{2}} S'(\xi) [(x-\xi)^2 + (z + a)^2]^{-1/2} d\xi \right\}_{z=a(x)} dx. \quad [9]$$

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The quantity in brackets can be evaluated for a slender body since it is a vertical dipole distribution:

$$\left\{ \frac{\partial}{\partial z} \int_{-\frac{L}{2}}^{\frac{L}{2}} S'(\xi) \left[(x-\xi)^2 + (z + (a(\xi))^2 \right]^{-1/2} d\xi \right\}_{z=a(x)}$$

$$= -\int_{-\frac{L}{2}}^{\frac{L}{2}} S'(\xi) \left[a(\xi) + a(x) \right] \left[(x-\xi)^2 + (a(\xi) + a(x))^2 \right]^{-3/2} d\zeta$$

$$\approx -2 a(x) S'(x) \int_{-\infty}^{\infty} \left[(x-\xi)^2 + 4 (a(x))^2 \right]^{-3/2} d\zeta$$

$$= - S'(x) / a(x) .$$

Thus, from [9], the vertical force is given by the integral

$$F_z = -\frac{1}{4\pi} \rho U^2 \int_{-\frac{L}{2}}^{\frac{L}{2}} [S'(x)]^2 \left\{ z_o^2 - [r_o(x)]^2 \right\}^{-1/2} dx,$$

or

$$F_z = - \pi \rho U^2 \int_{-\frac{L}{2}}^{\frac{L}{2}} [r_o(x) \; r_o'(x)]^2 \left\{ z_o^2 - [r_o(x)]^2 \right\}^{-1/2} dx. \qquad [10]$$

Similarly, for the moment,

$$M_y = \pi \rho U^2 \int_{-\frac{L}{2}}^{\frac{L}{2}} [r_o(x) \; r_o'(x)]^2 \left\{ z_o^2 - [r_o(x)]^2 \right\}^{-1/2} x \; dx. \qquad [11]$$

These two integrals are the principal results of the analysis. We note that for a body with longitudinal (fore-and-aft) symmetry the integral for the pitch moment M_y vanishes, as a consequence of the symmetrical flow which wil. then occur. Moreover, the vertical force F_z is always less than zero, in keeping with the fact that the flow velocity will be higher

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between the wall and the body, as compared with the corresponding value on the "outside" portion of the body. Thus there will always be a suction force tending to attract the body toward the wall and this force will increase monotonically to a finite limit as $z_o \to r_o$, or the body is brought in to the wall. We can conclude from the above integrals that for geometrically affine bodies, having the same cross-sectional distributions but different lengths, the force will be inversely proportional to the length whereas the moment will be independent of the length. Finally, we note that for bodies with flat ends, such as a right circular cylinder, the above integrals are divergent as a consequence of the "end effect" error in the slender-body theory.

The pitch moment is more difficult to discuss qualitatively since the integrand of [11] is not positive-definite, and the sign of the integral will depend critically on the distribution of the sectional area $S(x)$, and possibly also on the separation distance z_o. However, it seems likely that for bodies with full or blunt ends this integral will be dominated by the large values of $S'(x)$ at the two ends, and thus that the pitch moment will be positive (bow down) if the bow is blunt compared to the stern, and vice versa. (An analogous argument is common in naval architecture to explain the tendency of ships with full sterns to tend to head away from a bank when moving in restricted waters.)

THE VERTICAL FORCE ON A SPHEROID

As an illustration we shall consider the body to be a slender spheroid, of maximum radius r_{om}, so that

$$r_o(x) = r_{om} (1 - 4x^2/L^2)^{1/2}$$

For convenience we introduce the nondimensional parameter

$$\zeta = z_o/r_{om},$$

representing the nondimensionalized vertical height of the body axis above the bottom. Substituting in [10], we obtain the integral

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$$F_z = -16 \pi \rho U^2 \, r_{om}^{3} \, L^{-4} \int_{-\frac{L}{2}}^{\frac{L}{2}} (\zeta^2 - 1 + 4x^2/L^2)^{-1/2} \, x^2 \, dx$$

$$= -2 \pi \rho U^2 \, r_{om}^{3} \, L^{-1} \left\{ \zeta - \frac{1}{2}(\zeta^2 - 1) \log \left(\frac{\zeta + 1}{\zeta - 1} \right) \right\} \qquad [12]$$

$$= -\frac{4}{3} \pi \rho U^2 \, r_{om}^{3} \, L^{-1} \left[Q_0(\zeta) - Q_2(\zeta) \right]$$

where Q_n is the Legendre function of the second kind. For $\zeta \to 1$ the force tends to a limiting value of

$$F_{zmax} = -2\pi \rho U^2 \, r_{om}^{3} \, L^{-1}. \qquad [13]$$

In Figure 2 we show the ratio F_z/F_{zmax}, as a function of the height parameter $\zeta = z_o / r_{om}$.
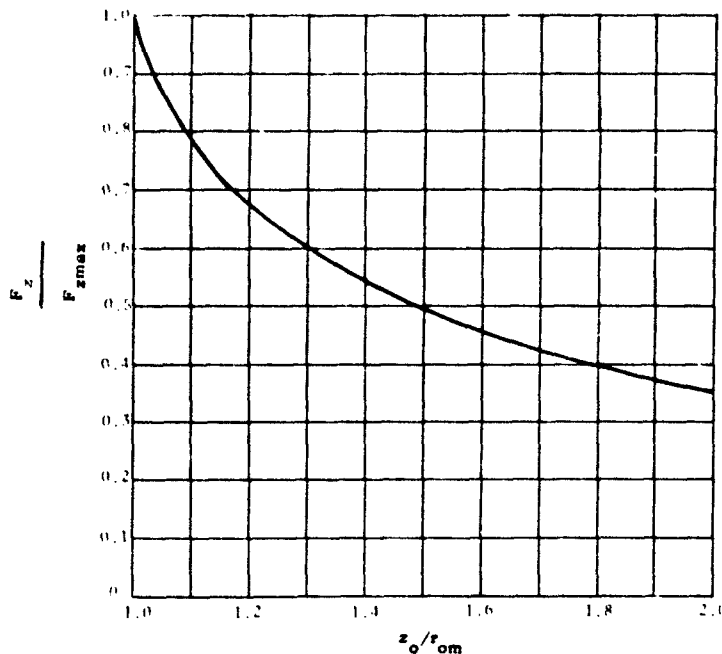

Figure 2 - Ratio of the Vertical Force to Its Limiting Maximum Value for a Slender Spheroid

ACKNOWLEDGMENT

9

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